HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Higher School Certificate
Trial Examination Term 3 2022

STUDENT NUMBER:_____

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–28)

- Attempt Questions 11–33
- Allow about 2 hour and 45 minutes for this section

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. What is the range of the function $y = -2\sqrt{16 - x^2}$?

   A. $(-\infty, -8]$

   B. $[-4, 0]$

   C. $[-8, 0]$

   D. $[-8, \infty)$

2. The circle $(x-2)^2 + (y+4)^2 = 9$ is enlarged by a factor of 2 with centre the origin. What is its resulting equation?

   A. $(x-1)^2 + (y+2)^2 = \frac{9}{4}$

   B. $(x-2)^2 + (y+4)^2 = 18$

   C. $(x-4)^2 + (y+8)^2 = 36$

   D. $(x-2)^2 + (y+4)^2 = 36$

3. The period of the function $f(x) = 2\tan(4x - \frac{\pi}{3})$, where $x \in \mathbb{R}$, is

   A. $\frac{\pi}{2}$

   B. $\frac{\pi}{4}$

   C. 2π

   D. π

Examination continues overleaf…

4. Which of the following is equal to $e^{-3\log_e x}$?

   A. $-3x$

   B. x^3

   C. e^{-3x}

   D. $\dfrac{1}{x^3}$

5. A particle, initially at the origin, moves in a straight line with velocity $v = 8 - 4t$ m/s. What is the total distance travelled by the particle in the first 4 seconds?

   A. 0 m

   B. 16 m

   C. 8 m

   D. 12 m

6. If $\int_2^5 f(x)\,dx = 4,$ which of the following is $\int_0^3 \big(3f(x+2)\big)\,dx$ equal to?

   A. 9

   B. 12

   C. 15

   D. 6

Examination continues overleaf…

7. In a group of 60 students, 24 have subscribed to SFLIX and 38 have subscribed to NETAN. If 12 students have subscribed to both SFLIX and NETAN, what is the probability that a student chosen at random has subscribed to neither SFLIX nor NETAN?

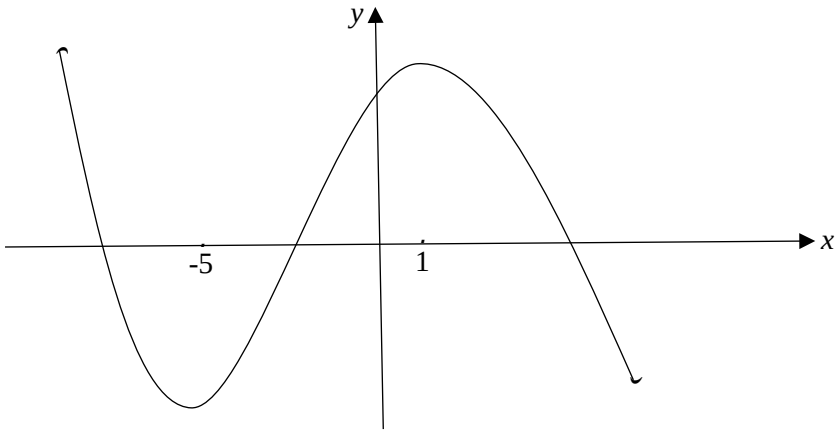
A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{19}{30}$

D. $\frac{11}{30}$

8. The graph of $y = f(x)$ is shown below. For which interval(s) is $f'(x) < 0$?



A. $(-\infty, -5] \cup [1, \infty)$

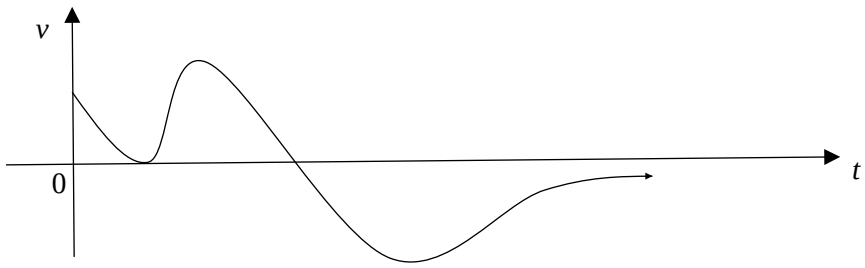
B. $(-5, 1)$

C. $(-\infty, -5) \cup (1, \infty)$

D. $[-5, 1]$

Examination continues overleaf…

9. A particle is moving along a straight line. The graph shows the velocity, v, of the particle for time $t \geq 0$.



How many times did the particle change direction?

A. 1

B. 2

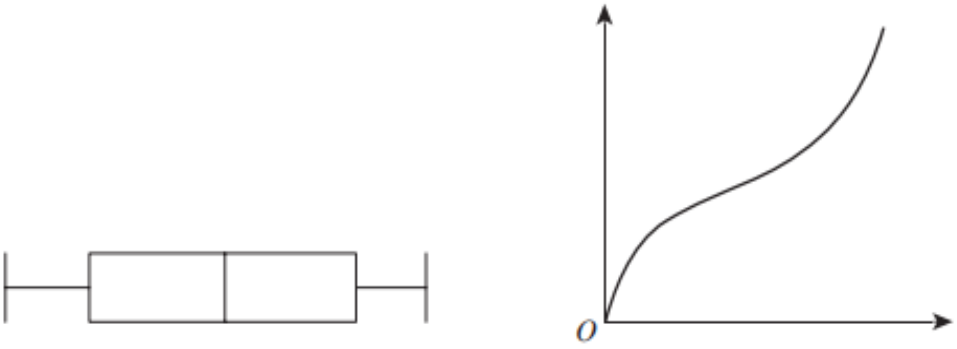
C. 3

D. 4

Examination continues overleaf…

10.

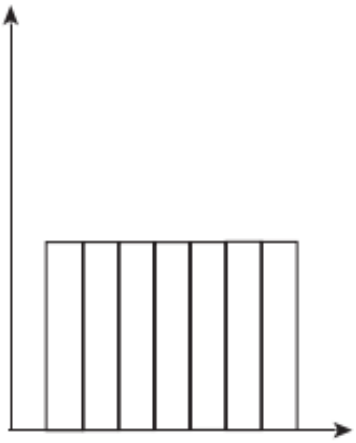
The box-plot and cumulative frequency diagram for a dataset are shown.



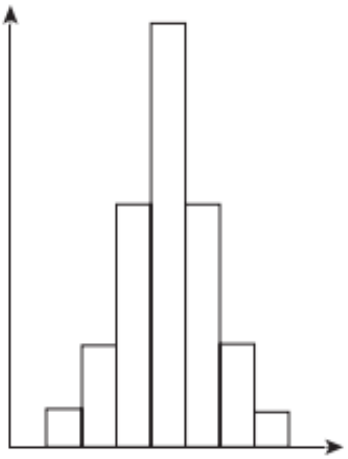
Box-plot *Cumulative frequency diagram*

Which of the following frequency histograms best represents the dataset?

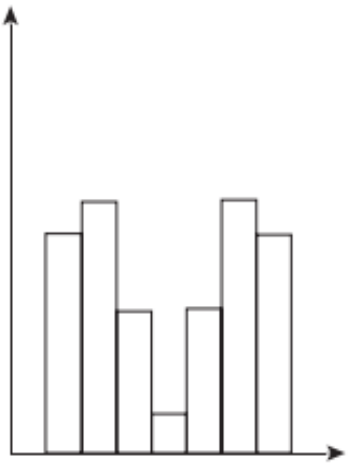
A.



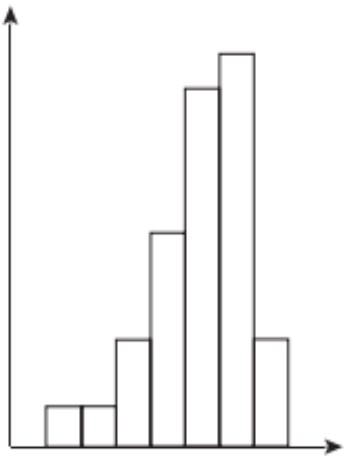
B.



C.



D.



End of Section I

Section II

90 marks
Attempt all questions
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (3 Marks)

Let $f(x) = (x+1)(x-1)$ and $g(x) = x^3$,

(a) Find the degree of $f(x) \times g(x)$. **1**

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(b) Is $f(x) \times g(x)$ odd or even? Justify your answer. **2**

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Examination continues overleaf…

Question 12 (4 Marks)

Differentiate the following:

(a) $y = \dfrac{2}{x^2 - 2x + 1}$ **2**

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(b) $y = \dfrac{e^{2x}}{x}$ **2**

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Examination continues overleaf…

Question 13 (4 Marks)

Find the following:

(a) $\int \frac{1}{(3x+2)^4} dx$ **2**

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(b) $\int \frac{2x^2}{x^3-1}\, dx$ **2**

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Examination continues overleaf…

Question 14 (3 Marks)

(a) Find $\frac{d\left(x^2 \ln x\right)}{dx}$. **2**

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(b) Hence, or otherwise, find the primitive of $2x + 4x \ln x$. **1**

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Question 15 (3 Marks)

Find, in general form, the equation of the normal to the curve $y = 2\sqrt{x}$ at the point on the curve where $x = 4$. **3**

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Examination continues overleaf

Question 16 (5 Marks)

The population of an endangered species of whale can be modelled by:

$$P = 350 - 10e^{0.05t}$$

where t is time in years from 1st January 2022 and P is the whale population.

(a) Determine the number of whales after 40 years from 1st January 2022. **1**

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(b) At what rate is the number of whales decreasing after 40 years from 1st January 2022? **2**
Give your answer correct to the nearest whole number.

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(c) <u>In what year</u> does the model predict that the species of whale will become extinct? **2**

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Examination continues overleaf…

Question 17 (6 Marks)

The acceleration of a particle moving on a straight line is given by $a = \sqrt{4t+16}$ m/s^2.

(a) What is the acceleration of the particle after 12 seconds? **1**

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(b) If the particle starts from rest, find its velocity after 12 seconds. **3**

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(c) Is the particle speeding up or slowing down after 12 seconds? Justify your answer. **2**

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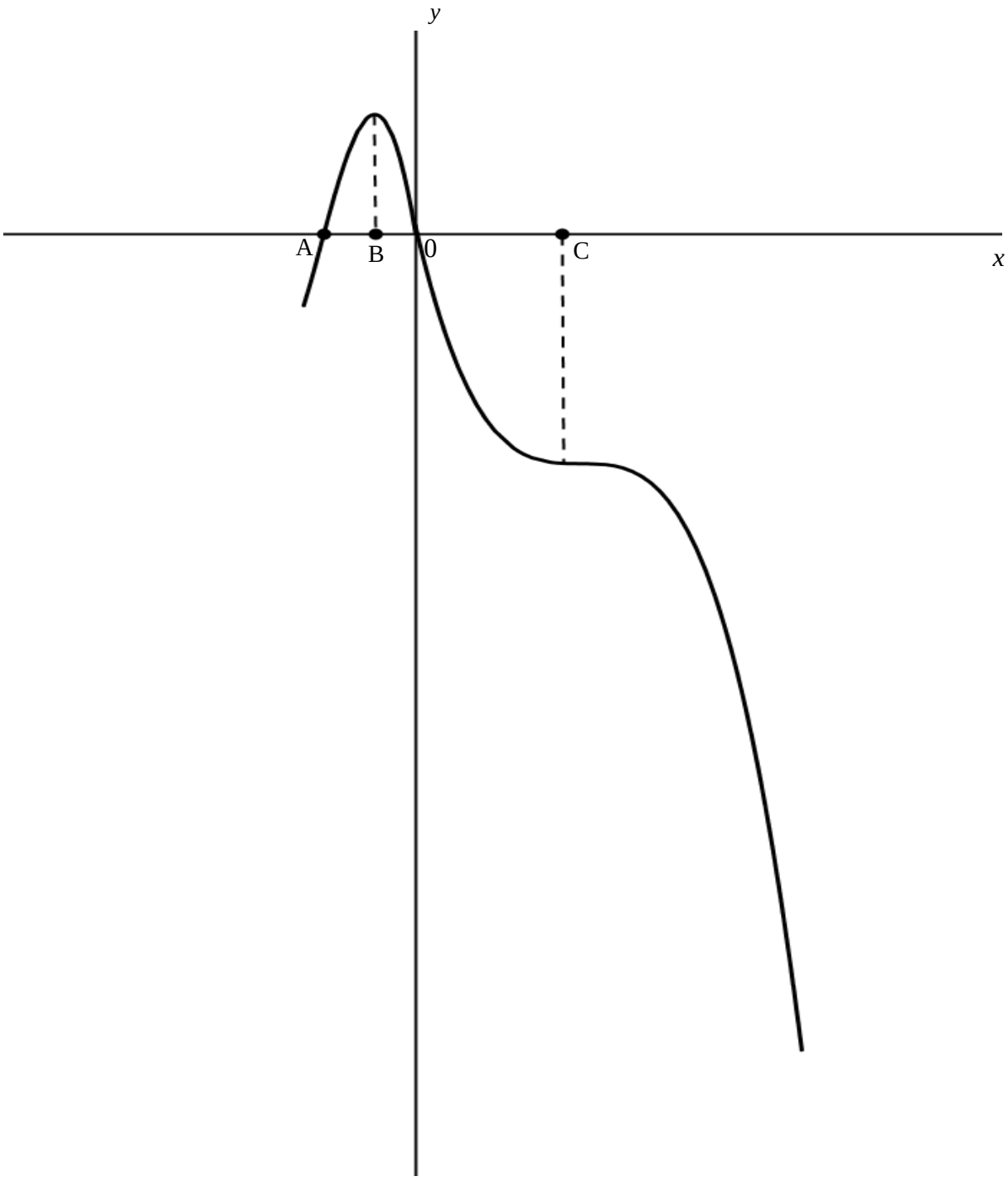
Examination continues overleaf…

Question 18 (3 Marks)

The graph of $y = f(x)$ is shown below. There is a maximum turning point at x = B, an inflexion point at 0, and a horizontal point of inflexion at x = C. **3**

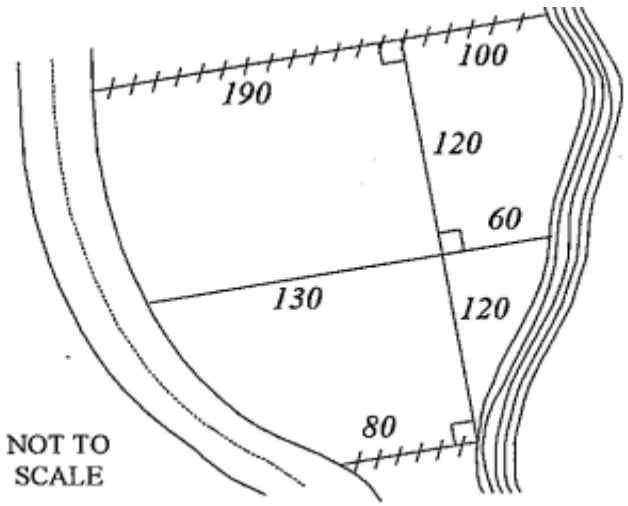
Draw the graph of $y = f'(x)$ on the same set of axes below.



Examination continues overleaf…

Question 19 (5 Marks)

The diagram below shows a farm property bordered by two straight fences, a river and a curved road.



(a) Using the trapezoidal rule and ALL the given measurements, find an approximation to the area of the farm property. All measurements are in metres. **3**

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(b) With the aid of the diagram above, <u>explain</u> whether the approximation found in part (a) above is an over-approximation or an under-approximation. **2**

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Examination continues overleaf…

Question 20 (3 Marks)

For the arithmetic series defined by $S_n = 2n^2 + 3n$, find the first, second and tenth terms. **3**

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Question 21 (2 Marks)

For the geometric series $4+8+16+\ldots$, write an equation for the sum of a geometric series given that $S_n = 508$ and solve the equation to find the value of n. **2**

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Examination continues overleaf….

Question 22 (5 Marks)

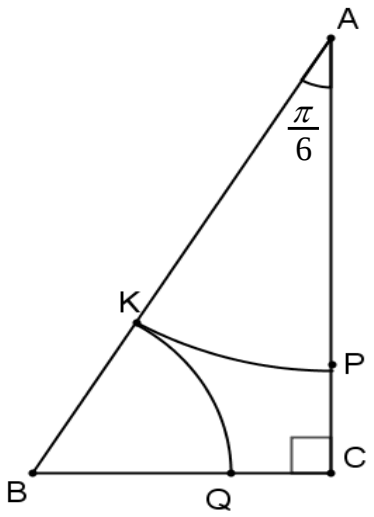
For the function $y = x^3 + \frac{3}{2}x^2 - 6x - 1$, find the stationary points and determine their nature and find any points of inflexion. Sketch the curve for $-4 \le x \le 2$. Also state the global minimum in the specified domain. Do NOT determine the x-intercepts of the curve. **5**

Examination continues overleaf…

Question 23 (6 Marks)

In the diagram, ΔABC is right angled at C, $\angle BAC = \frac{\pi}{6}$ and $AB = 5\text{cm}$.

KP is a circular arc with centre A and KQ is a circular arc with centre B.



NOT TO SCALE

If AK has length x cm, find the value of x, correct to one decimal place, for which the areas of the sectors AKP and BKQ are equal.

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Examination continues overleaf…

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Question 24 (2 Marks)

Find the value(s) of x when $\cot^2 x + \text{cosec}^2 x = 3$ for $0 \le x \le 2\pi$ **2**

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Question 25 (2 Marks)

Prove that $\dfrac{\cos\theta}{1-\sin\theta} - \tan\theta = \sec\theta$ **2**

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Examination continues overleaf…

Question 26 (6 Marks)

A game consists of a player throwing three fair coins. You lose \$4 if three heads appear and lose \$2 if two heads appear. You win \$2 if one head appears and win \$3 if no heads appear.

Let X be the amount you win or lose per game.

(a) Complete the probability distribution table below. **2**

x	-4	-2	2	3
$P(X = x)$				

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(b) Calculate $E(X)$ and hence determine the expected profit or loss if you played the game 500 times. **2**

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(c) Evaluate $P(X < -2 \mid X \leq -2)$ **2**

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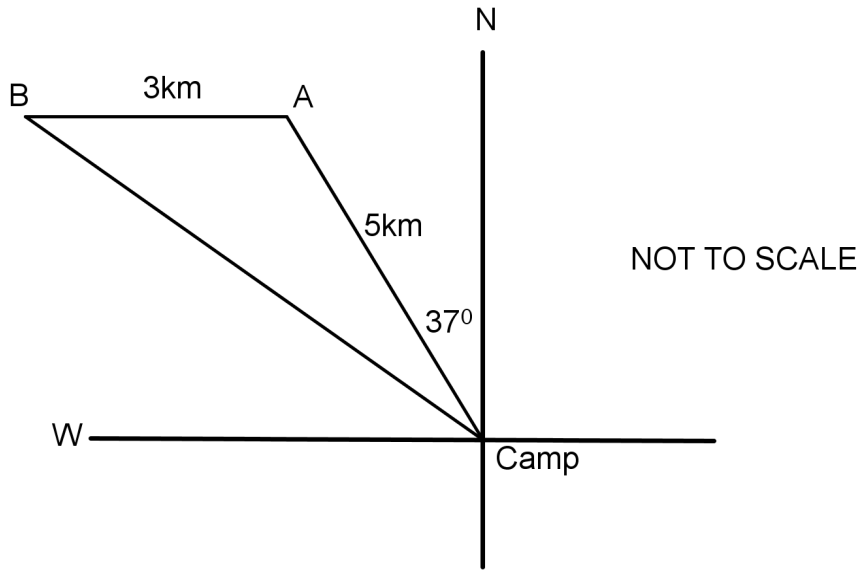
Examination continues overleaf…

Question 27 (4 Marks)

A hiker walks 5 km from camp to point A in a direction of $N37^{\circ}W$. They then walk 3 km due west to point B, as shown in the diagram below.



(a) What is the shortest distance between the hiker when at point B and the camp? **2**

(Answer to the nearest metre)

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(b) What is the bearing of the hiker at point B from the camp? **2**

(Answer to the nearest degree)

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Examination continues overleaf…

Question 28 (3 Marks)

Give a possible sequence of transformations that transforms $y = f(x)$ to $y = 3 \times f(2x - 4) + 1$.

Ensure that you use appropriate terminology in your response. **3**

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Examination continues overleaf…

Question 29 (6 Marks)

(a) Sketch the graph of $y = |2x + 5|$ on the lines provided below, clearly labelling the x and y intercepts. **2**

(b) Hence, or otherwise, solve $|4x + 10| + 2 = 8$. **3**

(c) By using the graph sketched in part (a) above, or otherwise, state the value(s) of m such that $|2x + 5| = m$ has no solutions. **1**

Examination continues overleaf…

Question 30 (3 Marks)

Suppose $f(x) = 2x^2 - x - 1$ and $g(x) = \cos x$. **3**

Find vertical asymptotes of $\dfrac{1}{f(g(x))}$ $\left(0 \le x \le \pi\right)$.

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Examination continues overleaf…

Question 31 (5 Marks)

Matt and Ingrid are playing a game. The dice they play with is four-sided and is marked with the numbers 1,2,3,4. They take turns throwing **two** dice. The game is won by the first player to throw a double four. Matt starts the game.

(a) Show that the probability that Ingrid wins the game on her first throw is $\frac{15}{256}$. **1**

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(b) What is the probability that Matt wins the game on the first or on the second throw? **2**

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(c) Find the probability that Matt eventually wins the game. **2**

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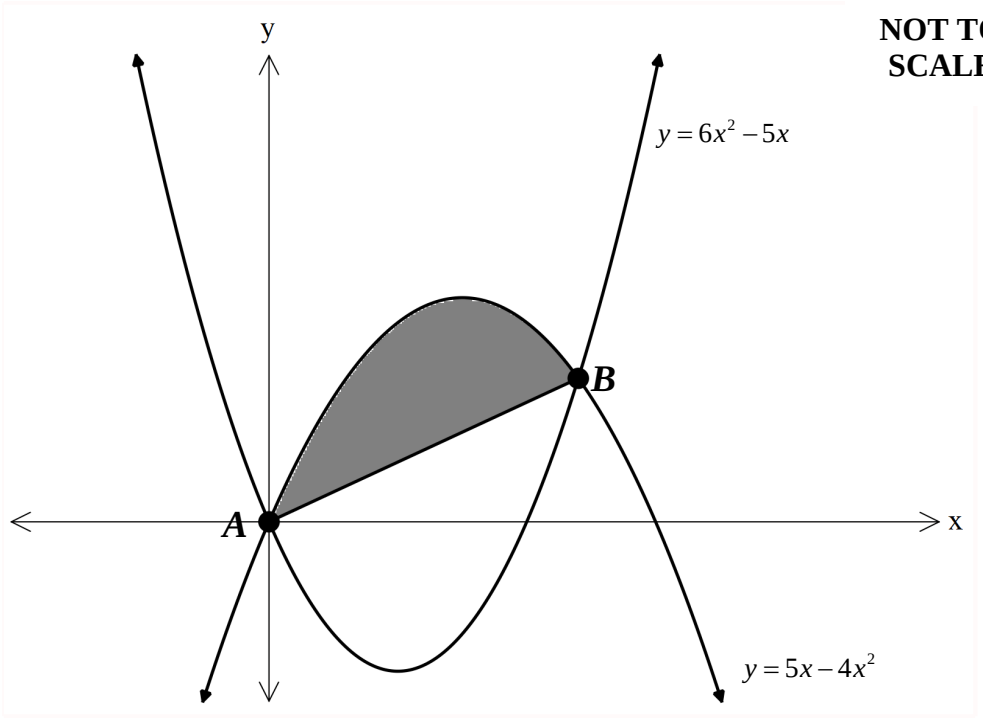
Examination continues overleaf…

Question 32 (3 Marks)

The diagram shows the curves with equation $y = 6x^2 - 5x$ and $y = 5x - 4x^2$. **3**
The curves intersect at *A* and *B* where *AB* is a chord.



NOT TO SCALE

Find the area of the shaded region.

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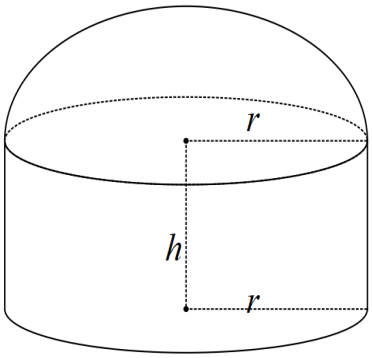
Examination continues overleaf…

Question 33 (6 Marks)

The figure below shows a hollow container consisting of a right circular cylinder of radius ***r*** cm and of height ***h*** cm joined to a hemisphere of radius ***r*** cm.



The cylinder is joined to the hemisphere at their open ends so that the resulting object is completely sealed.

(a) Given that volume of the container is exactly 360π cm^3, show clearly that the total surface area, S cm^2, of the exposed surfaces including the container's base is given by $S = \frac{5}{3}\pi r^2 + \frac{720\pi}{r}$. **3**

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Examination continues overleaf…

(b) Hence, show that $r = h$ when S is minimised. **3**

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End of Paper

HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Higher School Certificate
Trial Examination Term 3 2022

STUDENT NUMBER: Solutions

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- Working time – 3 hours
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10 marks
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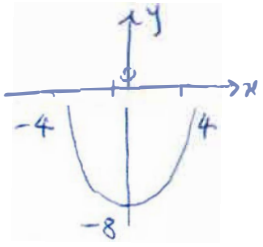


A. $(-\infty, -8]$

B. $[-4, 0]$

C. $[-8, 0]$

D. $[-8, \infty)$

2. The circle $(x-2)^2 + (y+4)^2 = 9$ is enlarged by a factor of 2 with centre the origin. What is its resulting equation?

$\left(\frac{x}{2} - 2\right)^2 + \left(\frac{y}{2} + 4\right)^2 = 9$

$\left(\frac{x-4}{2}\right)^2 + \left(\frac{y+8}{2}\right)^2 = 9$

$(x-4)^2 + (y+8)^2 = 36$

A. $(x-1)^2 + (y+2)^2 = \frac{9}{4}$

B. $(x-2)^2 + (y+4)^2 = 18$

C. $(x-4)^2 + (y+8)^2 = 36$

D. $(x-2)^2 + (y+4)^2 = 36$

3. The period of the function $f(x) = 2\tan(4x - \frac{\pi}{3})$, where $x \in \mathbb{R}$, is

$\text{period} = \frac{\pi}{n}$

$= \frac{\pi}{4}$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. 2π

D. π

Examination continues overleaf...

4. Which of the following is equal to $e^{-3\log_e x}$?

A. $-3x$

B. $\frac{1}{x}$

C. e^{-3x}

D. $\frac{1}{x^3}$

$e^{\log_e x^{-3}}$

$= x^{-3}$

$= \frac{1}{x^3}$

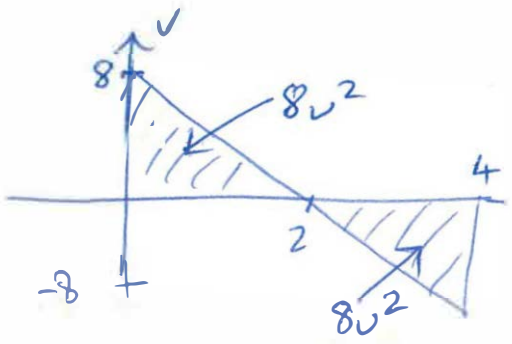
5. A particle, initially at the origin, moves in a straight line with velocity $v = 8 - 4t$ m/s. What is the total distance travelled by the particle in the first 4 seconds?

A. 0 m

B. 16 m

C. 8 m

D. 12 m



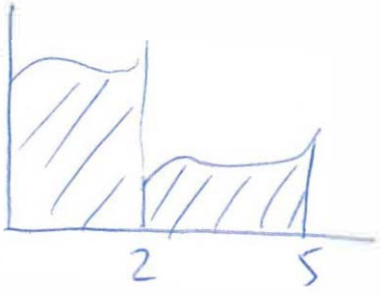
6. If $\int_2^5 f(x)\,dx = 4$, which of the following is $\int_0^3 (3f(x+2))\,dx$ equal to?

A. 9

B. 12

C. 15

D. 6



$= 3 \times 4$

$= 12$

Examination continues overleaf...

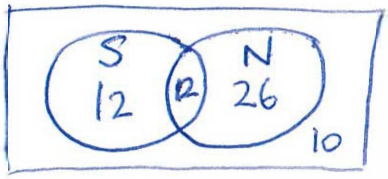
7. In a group of 60 students, 24 have subscribed to SFLIX and 38 have subscribed to NETAN. If 12 students have subscribed to both SFLIX and NETAN, what is the probability that a student chosen at random has subscribed to neither SFLIX nor NETAN?

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{19}{30}$

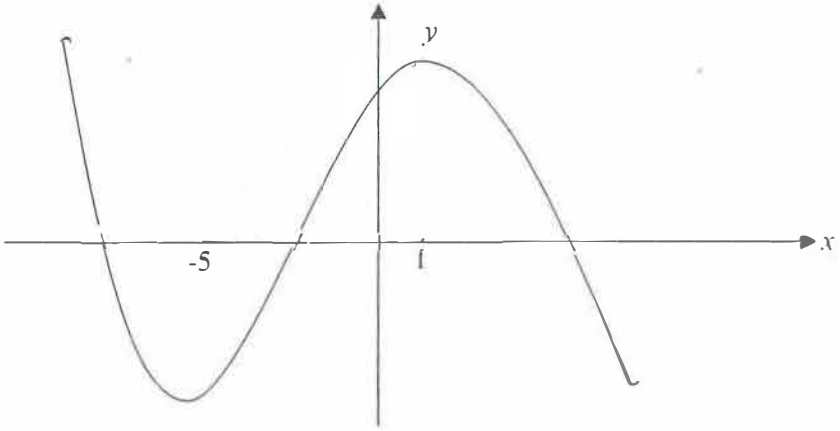
D. $\frac{11}{30}$



$12 + 12 + 26 = 50$

$\therefore$ 10 neither

8. The graph of $y = f(x)$ is shown below. For which interval(s) is $f'(x) < 0$?



A. $(-\infty, -5] \cup [1, \infty)$

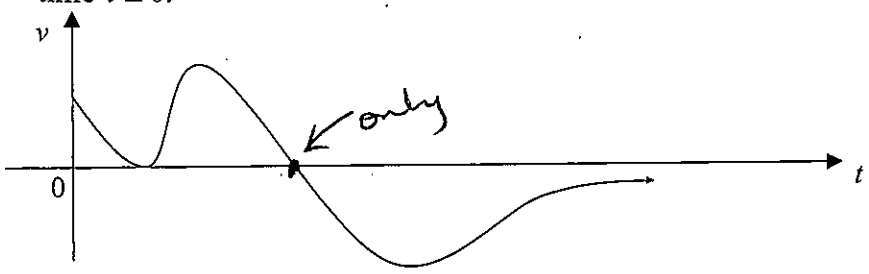
B. $(-5, 1)$

C. $(-\infty, -5) \cup (1, \infty)$

D. $[-5, 1]$

Examination continues overleaf...

9. A particle is moving along a straight line. The graph shows the velocity, v, of the particle for time $t \geq 0$.



How many times did the particle change direction?

A. 1

B. 2

C. 3

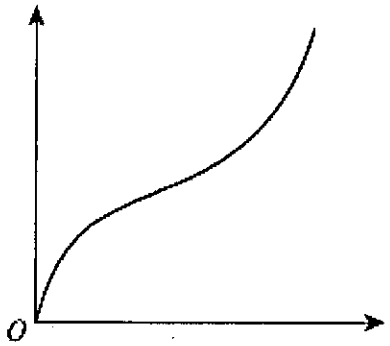
D. 4

Examination continues overleaf...

10.

The box-plot and cumulative frequency diagram for a dataset are shown.

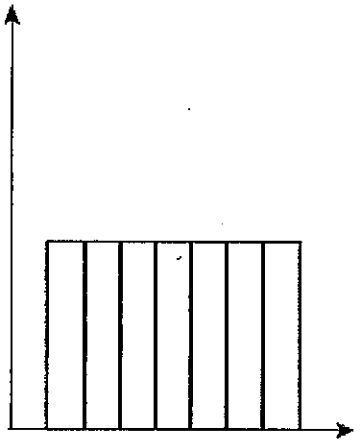
Box-plot



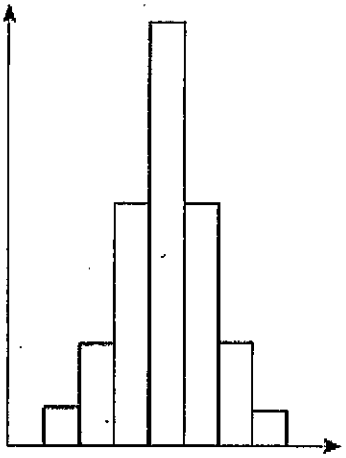
Cumulative frequency diagram

Which of the following frequency histograms best represents the dataset?

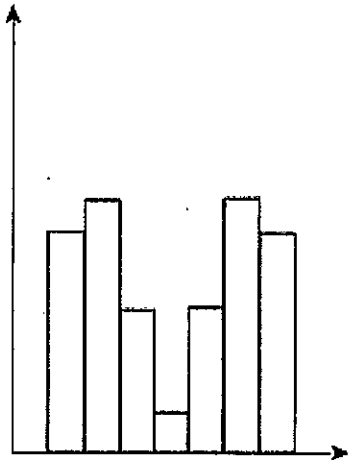
A.



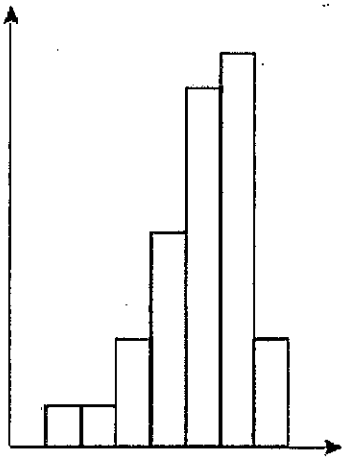
B.



C.



D.



End of Section I

Section II

90 marks
Attempt all questions
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (3 Marks)

Let $f(x)=(x+1)(x-1)$ and $g(x)=x^3$,

(a) Find the degree of $f(x)\times g(x)$. **1**

$f(x) = x^2 - 1$
$g(x) = x^3$

$\therefore f(x) \times g(x)$
$= (x^2-1)(x^3)$
$\therefore$ degree is 5

Generally well done.
Some student forgot what a degree is.

(b) Is $f(x)\times g(x)$ odd or even? Justify your answer. **2**

$f(x) \times g(x) = x^5 - x^3$
Let $F(x) = x^5 - x^3$
$F(-x) = (-x)^5 - (-x)^3$
$= -x^5 + x^3$
$= -(x^5 - x^3)$
$= -F(x)$
$\therefore F(-x) = -F(x)$
$\therefore$ function is odd.

Some students did not even did/it show substitution.

Examination continues overleaf...

Question 12 (4 Marks)

Differentiate the following:

(a) $y = \dfrac{2}{x^2 - 2x + 1}$ $= 2(x^2 - 2x + 1)^{-1}$ **2**

$y' = -2(x^2 - 2x + 1)^{-2} \times (2x - 2)$

$y' = \dfrac{-2(2x-2)}{(x^2 - 2x + 1)^2}$ generally well done.

$y' = \dfrac{4 - 4x}{(x^2 - 2x + 1)^2}$ ✓✓

or $y' = \dfrac{4(1-x)}{((x-1)^2)^2}$

$= \dfrac{-4(x-1)}{(x-1)^4} = \dfrac{-4}{(x-1)^3}$

(b) $y = \dfrac{e^{2x}}{x}$ **2**

$u = e^{2x}$ $\quad v = x$

$u' = 2e^{2x}$ $\quad v' = 1$

$y' = \dfrac{x(2e^{2x}) - e^{2x}}{x^2}$

$y' = \dfrac{e^{2x}(2x-1)}{x^2}$ generally well done

Examination continues overleaf…

Question 13 (4 Marks)

Find the following:

(a) $\int \frac{1}{(3x+2)^4} dx$ **2**

$= \int (3x+2)^{-4} dx$

$= \frac{(3x+2)^{-3}}{-3(3)} + C$

$= \frac{-1}{9(3x+2)^3} + C$

generally well done. some are not familiar with $\int (ax+b)^n dx$

(b) $\int \frac{2x^2}{x^3-1} dx$ **2**

$= \frac{2}{3} \int \frac{3x^2}{x^3-1} dx$

$= \frac{2}{3} \ln|x^3-1| + C$

some are not familiar with $\int \frac{f'(x)}{f(x)} dx$.

Examination continues overleaf...

Question 14 (3 Marks)

(a) Find $\dfrac{d\left(x^2 \ln x\right)}{dx}$. 2

$u = x^2 \qquad v = \ln x$

$u' = 2x \qquad v' = \dfrac{1}{x}$

$\dfrac{d}{dx}\left(x^2 \ln x\right) = (\ln x)(2x) + x^2\left(\dfrac{1}{x}\right)$

$2x \ln x + x$

Both (a) & (b) done very well.

(b) Hence, or otherwise, find the primitive of $2x + 4x \ln x$. 1

$\therefore \int (2x + 4x \ln x)\, dx$

$= 2\int x + 2x \ln x \, dx$

$= 2x^2 \ln x + C$ note: many did not have +C this was ignored

Question 15 (3 Marks)

Find, in general form, the equation of the normal to the curve $y = 2\sqrt{x}$ at the point on the curve where $x = 4$. 3

when $x = 4$, $y = 2\sqrt{4}$ $\therefore$ pt is $(4, 4)$

$y = 4$

$y = 2\sqrt{x}$

$y = 2x^{\frac{1}{2}}$

$y' = x^{-\frac{1}{2}}$

$y' = \dfrac{1}{\sqrt{x}}$ ① mark for correct differentiation.

$m_T = \dfrac{1}{\sqrt{4}}$ when $x = 4$

$\therefore m_N = -2$ ① using correct gradient.

$y - 4 = -2(x - 4)$ $\therefore 2x + y - 12 = 0$

$y - 4 = -2x + 8$ ① correct equation in the general form.

$y = -2x + 12$

Examination continues overleaf

Note: Many students do not know what the correct format 'general form' is!

Question 16 (5 Marks)

The population of an endangered species of whale can be modelled by:

$$P = 350 - 10e^{0.05t}$$

Where t is time in years from 1st January 2022.

(a) Determine the number of whales after 40 years. **1**

$P = 350 - 10e^{0.05(40)}$

$P = 276.1094$ Done well.

$\therefore P = 276$ whales

Accepted variations to this answer.

(b) At what rate is the number of whales decreasing after 40 years? **2**

Give your answer correct to ~~one decimal place~~ to the nearest whole number.

$\frac{dP}{dt} = -10e^{0.05t} \times 0.05$

$\frac{dP}{dt} = -\frac{1}{2}e^{0.05t}$

$= -1e^{0.05(40)}$

$= -3.69$

$\therefore$ the population is decreasing at a rate of 4 whales/yr.

Many students did not actually answer the question. Asking for rate

(c) In what year does the model predict that the species of whale will become extinct? **2**

$0 = 350 - 10e^{0.05t}$

$10e^{0.05t} = 350$

$e^{0.05t} = 35$

* Need to answer the question.

$0.05t \ln 35$

$t = \frac{\ln 35}{0.05}$

$t = 71.10696123$

$\therefore$ the whale population will become extinct during the year 2093.

Also accepted 2094.

Examination continues overleaf...

Question 17 (6 Marks)

The acceleration of a particle moving on a straight line is given by $a = \sqrt{4t+16}$ m/s^2.

(a) What is the acceleration of the particle after 12 seconds? 1

Done well in here
Some students wrote ±8!

$a = \sqrt{4(12)+16}$

$a = 8 m/s^2$

(b) If the particle starts from rest, find its velocity after 12 seconds. 3

$a = (4t+16)^{\frac{1}{2}}$

$v = \int (4t+16)^{\frac{1}{2}} dt$

$v = \frac{(4t+16)^{\frac{3}{2}}}{\frac{3}{2}\times 4} + c$

$v = \frac{(4t+16)^{\frac{3}{2}}}{6} + c$

$v = 0, t = 0$

$0 = \frac{16^{\frac{3}{2}}}{6} + c$

$c = -\frac{32}{3}$

$\therefore v = \frac{\sqrt{(4t+16)^3}}{6} - \frac{32}{3}$

$v_{12} = \frac{\sqrt{(4(12)+16)^3}}{6} - \frac{32}{3}$

Poorly done.

$\therefore v = \frac{224}{3}$ m/s

(c) Is the particle speeding up or slowing down after 12 seconds? Justify your answer. 2

The particle is speeding up because both the acceleration & velocity are positive

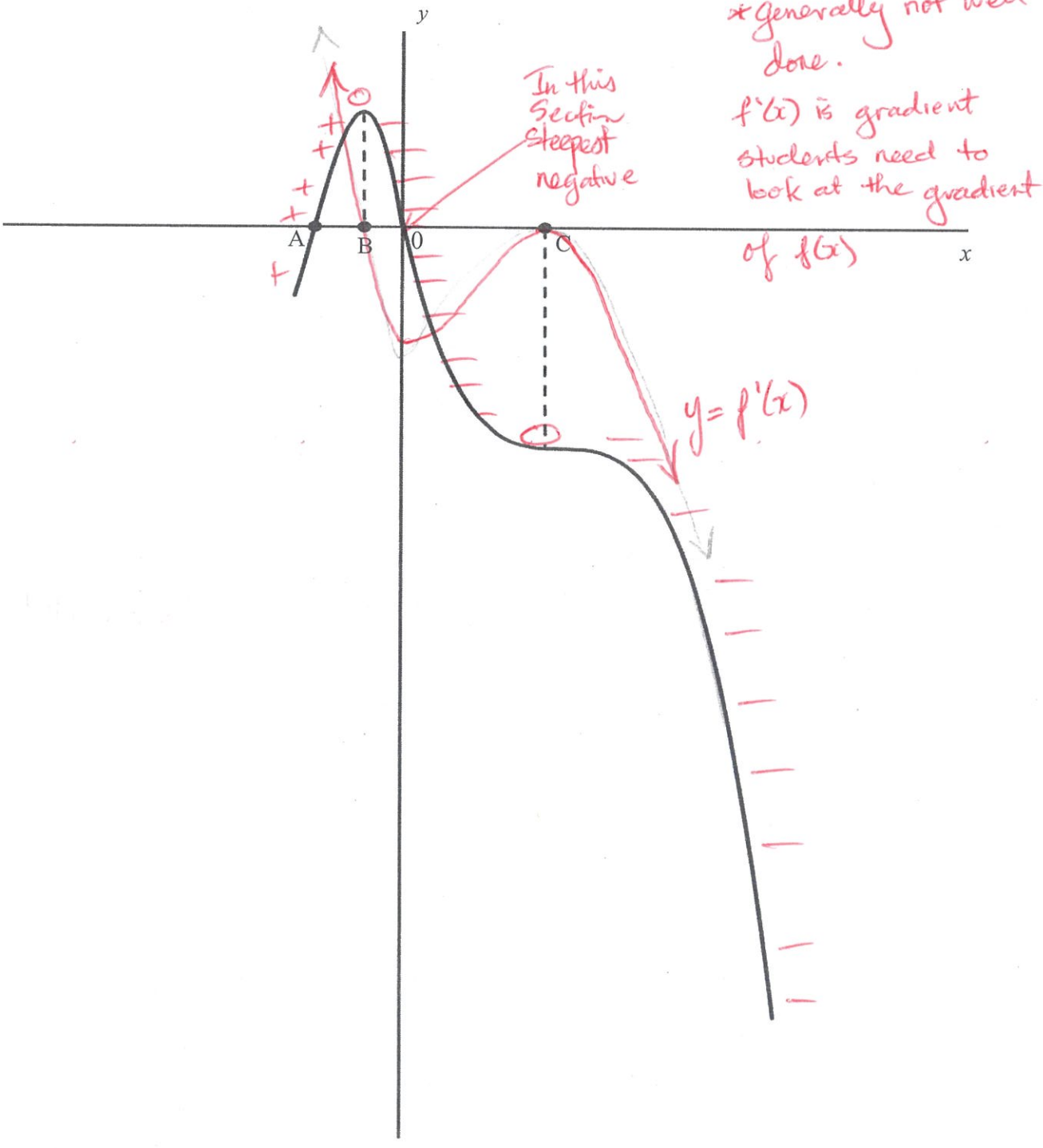
OK! Students needed to comment on velocity + acceleration.

Examination continues overleaf...

Question 18 (3 Marks)

The graph of $y = f(x)$ is shown below. There is a maximum turning point at $x = \text{B}$, an inflexion point at 0, and a horizontal point of inflexion at $x = \text{C}$. **3**

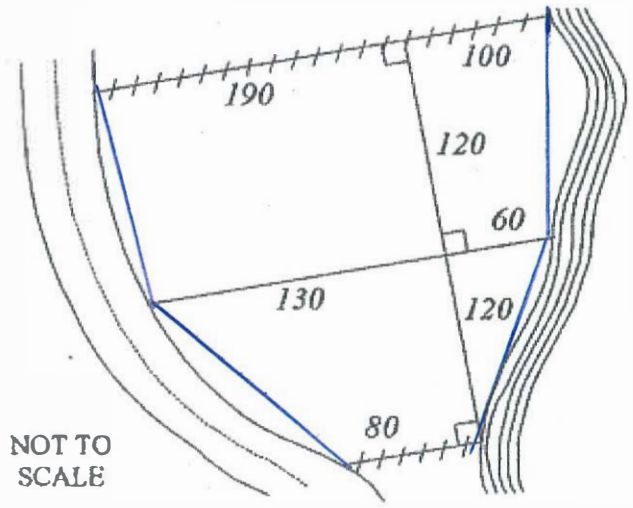
Draw the graph of $y = f'(x)$ on the same set of axes below.



Examination continues overleaf...

Question 19 (5 Marks)

The diagram below shows a farm property bordered by two straight fences, a river and a curved road.



(a) Using the trapezoidal rule and ALL the given measurements, find an approximation to the area of the farm property. **3**

$A \approx \frac{120}{2}[80 + 290 + 2(190)]$

$A \approx 60(750)$

Done well!

$A \approx 45\,000\,m^2$

(b) With the aid of the diagram above, explain whether the approximation found in part (a) above is an over-approximation or an under-approximation. **2**

By inserting straight lines around the river & road, we can see that the area found above will be an under-approximation as there are areas between the road & the river not calculated. Well done!

Examination continues overleaf...

Question 20 (3 Marks)

For the arithmetic series defined by $S_n = 2n^2 + 3n$, find the first, second and tenth terms. 3

$S_1 = T_1 = 2(1)^2 + 3(1)$

$T_1 = 5$

$S_2 = T_1 + T_2$

$S_2 = 2(2)^2 + 3(2)$

$= 8 + 6$

$= 14$

$\therefore 14 = 5 + T_2$

$\therefore T_2 = 9$

5, 9, ...

$a = 5, \ d = 4$

$\therefore T_{10} = a + (n-1)d$

$= 5 + 9 \times 4$

$\therefore T_{10} = 41$

Done well by most students

Some students confused T_2 with S_2

$T_2 \neq S_2 = 14$

Some used T_2 as d

$T_2 = 9, \ d \neq 9$

Question 21 (2 marks)

For the geometric series $4 + 8 + 16 + \ldots$, write an equation for the sum of a geometric series given that $S_n = 508$ and solve the equation to find the value of n. 2

$a = 4, \ r = 2$

$S_n = \frac{4(2^n - 1)}{2 - 1}$

$\therefore 508 = 4(2^n - 1)$

$127 = 2^n - 1$

$128 = 2^n$

$\therefore n = 7$

Some students need to revise logs

Examination continues overleaf...

Question 21 (5 Marks)

For the function $y = x^3 + \frac{3}{2}x^2 - 6x - 1$, find the stationary points and determine their nature and find any points of inflexion. Sketch the curve for $-4 \le x \le 2$. Also state the global minimum in the specified domain. Do NOT determine the x-intercepts of the curve. **5**

$y = x^3 + \frac{3}{2}x^2 - 6x - 1$

$y' = 3x^2 + 3x - 6$

$y' = 3(x^2 + x - 2)$

$y' = 3(x+2)(x-1)$

$0 = 3(x+2)(x-1)$ for stat. pts.

$x = -2, \quad x = 1$ ① for finding $x=-2$, $x=1$

$y = 9 \quad y = -4.5$

∴ stat pts are at $(-2, 9)$ & $(1, -4.5)$

$y'' = 6x + 3$

when $x = -2$, $y'' = -12 + 3$

$y'' = -9 < 0$ ∴ max tp at $(-2, 9)$ (concave down)

when $x = 1$, $y'' = 6 + 3$

$y'' = 9 > 0$ ∴ min tp at $(1, -4.5)$ (concave up) ① for determining nature and finding y values

Possible point of inflexion at $y'' = 0$

$6x + 3 = 0$

$6x = -3$

$x = -\frac{1}{2}, \quad y = 2\frac{1}{4}$

x	-1	$-\frac{1}{2}$	0
y''	-3	0	3

∩ change in concavity ∪

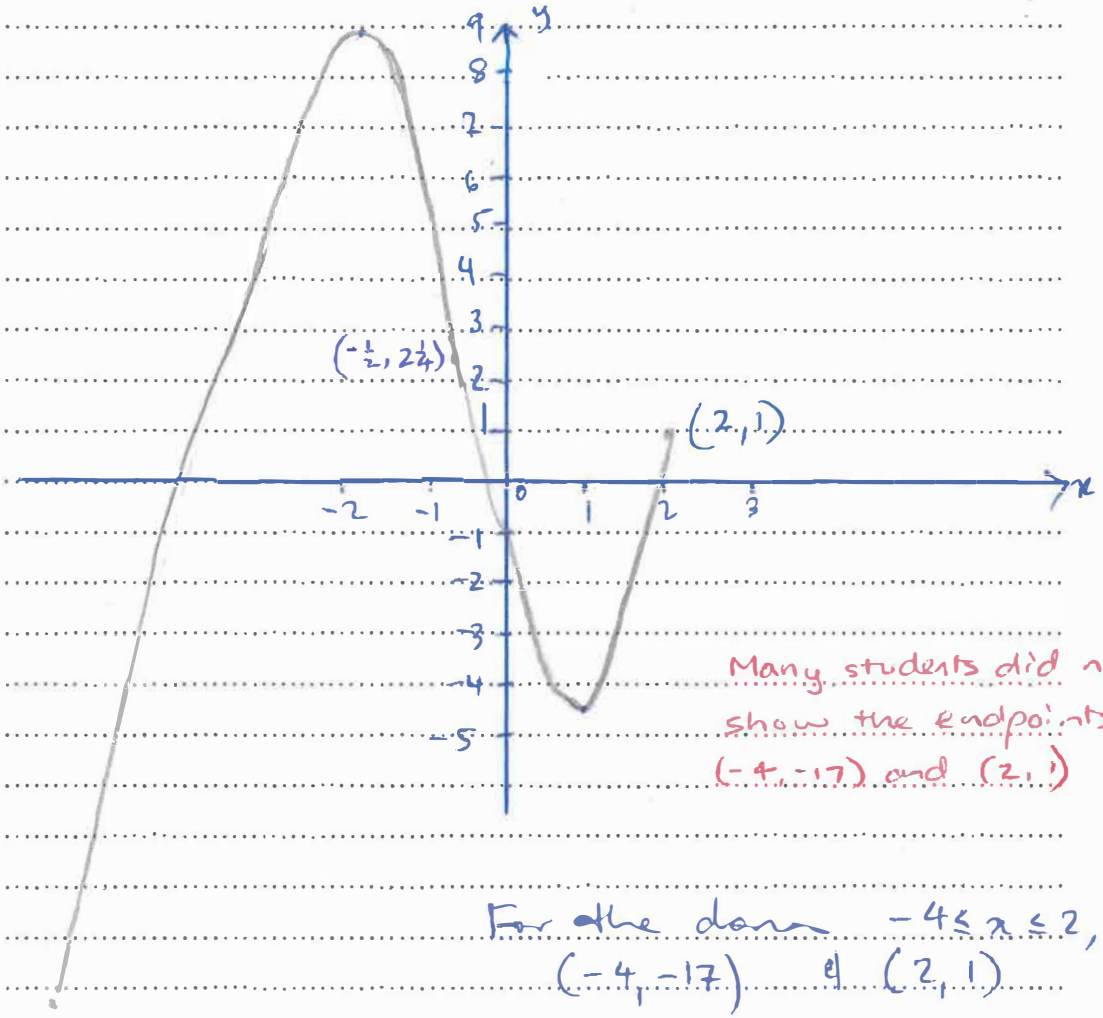
∴ $(-\frac{1}{2}, 2\frac{1}{4})$ is a point of inflexion.

① for point of inflexion and checking for change in concavity

N.B This is NOT a horizontal point of inflexion because it is not a stationary point.

Many students made an error with their sketch

Examination continues overleaf…



Many students did not show the endpoints (-4, -17) and (2, 1)

For the domain $-4 \leq x \leq 2$, $(-4, -17)$ & $(2, 1)$

$(-4, -17)$ ** ∴ global minimum is -17.

Extremely poorly done

Global/Absolute maximum or minimum are looking for the y value only

NOT the point (-4, -17) and not $y = -17$

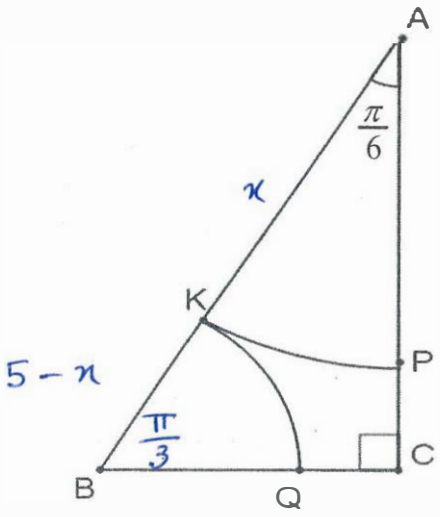
Just, global minimum is -17.

Examination continues overleaf...

Question 23 (6 Marks)

In the diagram, ΔABC is right angled at C, $\angle BAC = \frac{\pi}{6}$ and $AB = 5\text{cm}$.

KP is a circular arc with centre A and KQ is a circular arc with centre B.



NOT TO SCALE

If AK is of length x cm, find the value of x (correct to one decimal place) for which the areas of the sectors AKP and BKQ are equal.

Area $_{AKP} = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} x^2 \times \frac{\pi}{6}$

$= \frac{\pi x^2}{12}$ ① for finding area of sector

① for BK = 5 − x, $\angle ABC = \frac{\pi}{3}$

Area $_{BKQ} = \frac{1}{2}(5-x)^2 \frac{\pi}{3}$

$= \frac{\pi}{6}(5-x)^2$ ①

For the areas to be equal

$\frac{\pi x^2}{12} = \frac{\pi}{6}(5-x)^2$

$\frac{x^2}{12} = \frac{(5-x)^2}{6}$

$\therefore x^2 = 2(5-x)^2$

$x^2 = 2(25 - 10x + x^2)$

$x^2 - 20x + 50 = 0$ ①

Examination continues overleaf...

$x = \frac{20 \pm \sqrt{400-200}}{2} = \frac{20 \pm \sqrt{200}}{2}$ ← several students made an error here

$x = \frac{20 \pm 10\sqrt{2}}{2} \neq 10 \pm \sqrt{200}$

$x = 10 \pm 5\sqrt{2}$

$x = 2.928932, \quad x = 17.071$ ① for correct x values

but $0 < x < 5$

$\therefore x = 2.9$ (1 dp) ① for rejecting x values outside $0 < x < 5$

Question 24 (2 Marks)

Find the value(s) of x when $\cot^2 x + \operatorname{cosec}^2 x = 3$ for $0 \leq x \leq 2\pi$ 2

all four quad.

$\cot^2 x + 1 = \operatorname{cosec}^2 x$ ← trig identity

$\cot^2 x + \cot^2 x + 1 = 3$

$2\cot^2 x = 2$

$\cot^2 x = 1$

$\tan^2 x = 1 \quad \tan x = \pm 1$

Not well done

acute $x = \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Question 25 (2 Marks)

Prove that $\frac{\cos\theta}{1-\sin\theta} - \tan\theta = \sec\theta$ 2

LHS $= \frac{\cos\theta}{1-\sin\theta} - \tan\theta$

generally well done

$= \frac{\cos\theta}{1-\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta - \sin\theta(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$

$= \frac{\cos^2\theta - \sin\theta + \sin^2\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{1-\sin\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{1}{\cos\theta} = \sec\theta = \text{RHS}$

Examination continues overleaf...

Question 26 (6 Marks)

A game consists of a player throwing three fair coins. You lose \$4 if three heads appear and lose \$2 if two heads appear. You win \$2 if one head appears and win \$3 if no heads appear.

Let X be the amount you win or lose per game.

(a) Complete the probability distribution table below. **2**

x	-4	-2	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

HHH HTT TTH
HHT THH TTT
HTH THT

generally well done

(b) Calculate $E(X)$ and hence determine the expected profit or loss if you played the game 500 times. **2**

$E(X) = -4\left(\frac{1}{8}\right) - 2\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right)$

$= -\frac{1}{8}$

generally well done

Expected value $= 500 \times \left(-\frac{1}{8}\right)$

$= \$62.50$ loss.

(c) Evaluate $P(X < -2 \mid X \leq -2)$ **2**

$= \frac{\frac{1}{8}}{\left(\frac{1}{8} + \frac{3}{8}\right)}$

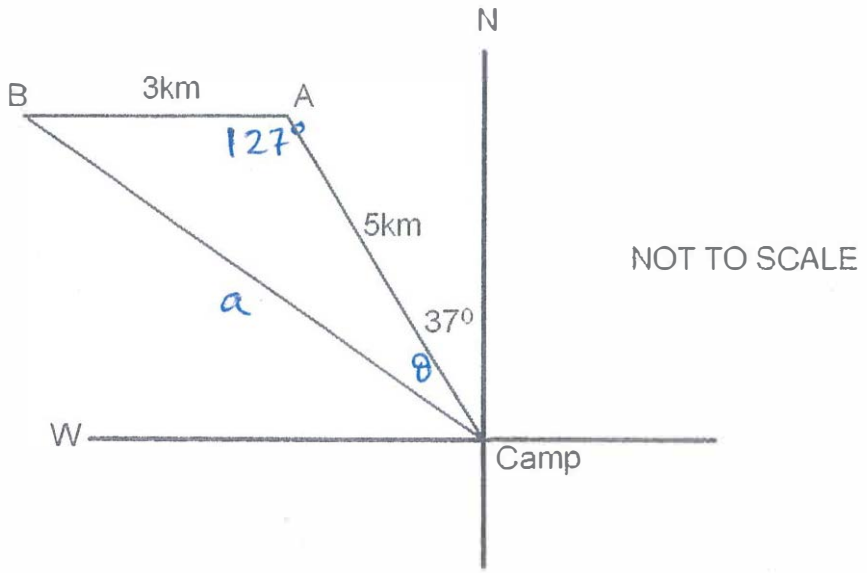
$= \frac{1}{4}$

mostly well done

Examination continues overleaf...

Question 27 (4 Marks)

A hiker walks 5 km from camp in a direction of $N37^{\circ}W$ to point A. They then walk 3 km due west to point B as shown in the diagram below.



(a) What is the shortest distance between the hiker when at point B and the camp? **2**

(Answer to the nearest metre)

$a^2 = 3^2 + 5^2 - 2(3)(5)\cos 127^{\circ}$

$a^2 = 52.05445\ldots$

$a = 7.214877\ldots$

$\therefore a = 7.215\text{ km}$

The majority of students did not do this correctly, and rounded to 7m ?? Think!

(b) What is the bearing of the hiker at point B from the camp? **2**

(Answer to the nearest degree)

$\frac{\sin\theta}{3} = \frac{\sin 127^{\circ}}{7.215}$

$\sin\theta = \frac{3\sin 127^{\circ}}{7.215}$

$\theta = 19^{\circ}24'$

$\theta = 19^{\circ}$ (nearest degree)

$\therefore$ bearing of B from Camp is 304° or $N56^{\circ}W$

Mostly well done

A lot of follow on errors from above

Examination continues overleaf...

Question 28 (3 Marks)

Give a possible sequence of transformations that transforms $y = f(x)$ to $y = 3 \times f(2x-4)+1$.

Ensure that you use appropriate terminology in your response. 3

$y = 3 \times f(2x-4)+1$

$y-1 = 3 \times f(2(x-2))$

$\frac{y-1}{3} = f\left(\frac{x-2}{\frac{1}{2}}\right)$

Vertically — Horizontally

Dilate vertically by a factor of 3 and then shift vertically up by 1 unit — Dilate horizontally by a factor of $\frac{1}{2}$ and then shift right by 2 units.

Lots of confusion

or

Shift vertically up by $\frac{1}{3}$ unit and then dilate vertically by a factor of 3.

or

Shift right by 4 units and then dilate horizontally by a factor of $\frac{1}{2}$.

(compress by a factor of 2 is acceptable)

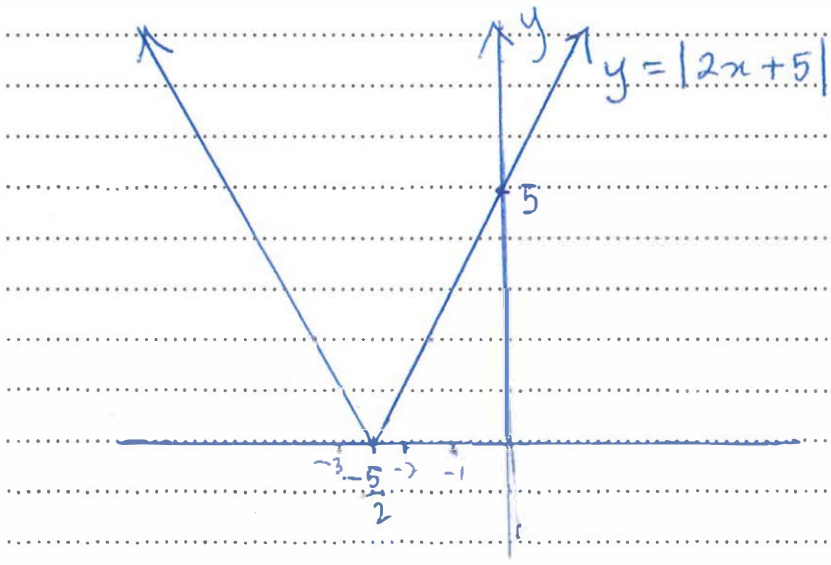
It does not matter which order the vertical or horizontal transformations are performed but within the vertical and horizontal transformation, the order does matter.

See 12 Camb Adv p103
and/or 12 Camb Ext1 p115

Examination continues overleaf...

Question 29 (6 Marks)

(a) Sketch the graph of $y = |2x+5|$ on the lines provided below, clearly labelling the x and y intercepts. **2**



Well done! Some students didn't label the diagram with x & y-int.

(b) Hence, or otherwise, solve $|4x+10|+2=8$. **3**

$|4x+10| = 6$

$2|2x+5| = 6$

$|2x+5| = 3$

$2x+5 = 3$ $\quad$ $2x+5 = -3$

$2x = -2$ $\quad$ $2x = -8$

$x = -1$ $\quad$ $x = -4$ Well done!

(c) By using the graph sketched in part (a) above, or otherwise, state the value(s) of m such that $|2x+5| = m$ has no solutions. **1**

$m < 0$ Well done!

Examination continues overleaf...

Question 30 (3 Marks)

Suppose $f(x) = 2x^2 - x - 1$ and $g(x) = \cos x$. **3**

Find vertical asymptotes of $\dfrac{1}{f(g(x))}$ $(0 \le x \le \pi)$.

$$\frac{1}{2\cos^2 x - \cos x - 1}$$

For vertical asymptotes, $2\cos^2 x - \cos x - 1 = 0$

let $u = \cos x$

$2u^2 - u - 1 = 0$

$(2u + 1)(u - 1) = 0$

$u = -\frac{1}{2}$, $u = 1$

$\cos x = -\frac{1}{2}$, $\cos x = 1$

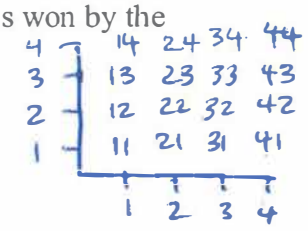
$\therefore x = \frac{2\pi}{3}$, $x = 0$

Generally well done. some student didn't check the domain for x.

Examination continues overleaf...

Question 31 (5 Marks)

Matt and Ingrid are playing a game. The dice they play with is four-sided and is marked with the numbers 1,2,3,4. They take turns throwing **two** dice. The game is won by the first player to throw a double four. Matt starts the game.



(a) What is the probability that Ingrid wins the game on her first throw? **1**

$P(\text{Ingrid wins}) = \frac{15}{16} \times \frac{1}{16}$

Matt loses ↑ Ingrid wins ↑

Also $P(\text{Ingrid wins}) = \left(\frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$

(b) What is the probability that Matt wins the game on the first or on the second throw? **2**

$P(\text{Matt wins 1st or 2nd})$

$= \frac{1}{16} + \frac{15}{16} \times \frac{15}{16} \times \frac{1}{16}$

$= \frac{1}{16} + \frac{15^2}{16^3}$

$= \frac{241}{256}$ $\frac{481}{4096}$

(c) Find the probability that Matt eventually wins the game. **2**

$S = \frac{1}{16} + \frac{15^2}{16^3} + \frac{15^4}{16^5} + \ldots$

$S = \frac{1}{16}\left[1 + \frac{15^2}{16^2} + \frac{15^4}{16^4} + \ldots\right]$

$a = 1 \qquad = \frac{15^2}{16^2}$

$S = \frac{1}{16}$

$1 - \frac{15^2}{16^2}$

$S_\infty = \frac{16}{31}$

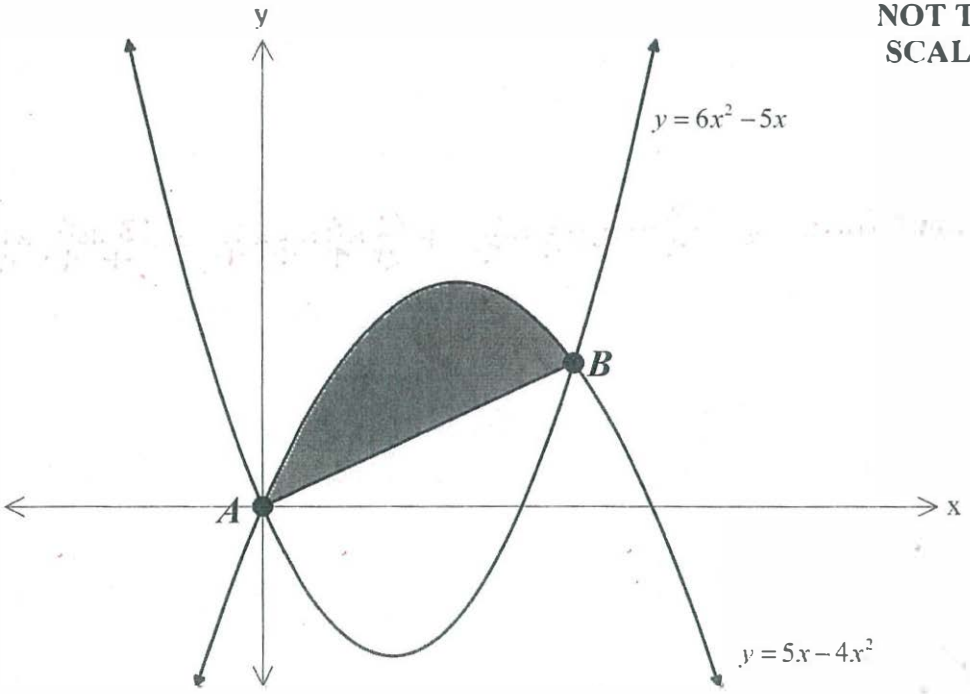
Many did not realise that the series was a limiting Sum.

$P(\text{Matt}) \text{ wins} = \frac{16}{31}$

Examination continues overleaf...

Question 31 (3 Marks)

The diagram shows the curves with equation $y = 6x^2 - 5x$ and $y = 5x - 4x^2$. **3**
The curves intersect at A and B.



NOT TO SCALE

Find the area of the shaded region.

Find equation of chord AB.

$6x^2 - 5x = 5x - 4x^2$ when $x=1$, $y=1$

$10x^2 - 10x = 0$ $\therefore m = 1$

$10x(x-1) = 0$ Equation $y = x$

$\therefore x = 0$ or $x = 1$

$$\therefore A = \int_0^1 [(5x - 4x^2) - x]\,dx$$

$$= \int_0^1 [4x - 4x^2]\,dx$$

$$= \left[\frac{4x^2}{2} - \frac{4x^3}{3}\right]_0^1$$

$$= \left[2(1)^2 - \frac{4}{3}(1)^3\right] - [0]$$

$$= 2 - \frac{4}{3}$$

$$= \frac{2}{3}\ u^2$$

common mistake was assuming areas between two curves!

$\int [5x - 4x^2 - (6x^2 - 5x)]\,dx$.

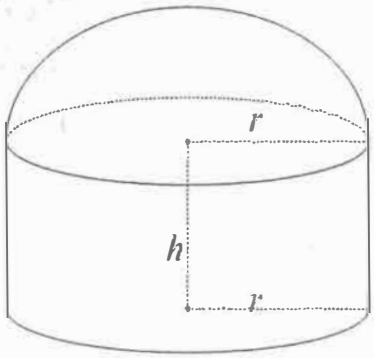
$= \frac{5}{3}u^2$

this was given (2).

Examination continues overleaf...

Question 33 (6 Marks)

The figure below shows a hollow container consisting of a right circular cylinder of radius r cm and of height h cm joined to a hemisphere of radius r cm.



The cylinder is open on one of the circular ends and the hemisphere is also open on its circular base. The cylinder is joined to the hemisphere at their open ends so that the resulting object is completely sealed.

(a) Given that volume of the container is exactly 360π cm^3, show clearly that the total surface area of the container, S cm^2, is given by $S = \frac{5}{3}\pi r^2 + \frac{720\pi}{r}$. 3

$V = \pi r^2 h + \frac{1}{2} \times \frac{4}{3}\pi r^3$

$360\pi = \pi r^2 h + \frac{2}{3}\pi r^3$

$360 = r^2 h + \frac{2}{3} r^3$

$r^2 h = 360 - \frac{2}{3} r^3$

$h = \frac{360}{r^2} - \frac{2}{3} r$ —(1)

Many student could find h in terms of r.

$S = \pi r^2 + 2\pi r h + \frac{1}{2} \times 4\pi r^2$

Some students were not able to find the correct surface area

$S = 3\pi r^2 + 2\pi r\left(\frac{360}{r^2} - \frac{2}{3} r\right)$ from (1)

$S = 3\pi r^2 + \frac{720\pi}{r} - \frac{4\pi}{3} r^2$

$S = \frac{5}{3}\pi r^2 + \frac{720\pi}{r}$

Examination continues overleaf...

(b) Hence, show that $r = h$ when S is minimised. $S = \frac{5}{3}\pi r^2 + 720\pi r^{-1}$ 3

$S' = \frac{10}{3}\pi r - 720\pi r^{-2}$

$S' = \frac{10}{3}\pi r - \frac{720\pi}{r^2} = 0$ for stat. pts

$\frac{10\pi r}{3} = \frac{720\pi}{r^2}$

$\frac{\pi r}{3} = \frac{72\pi}{r^2}$

$\frac{r}{3} = \frac{72}{r^2}$

$r^3 = 216$

$r = 6$

$S'' = \frac{10}{3}\pi + 1440\pi r^{-3}$

$S'' \frac{10}{3}\pi + \frac{1440\pi}{r^3} > 0$ for $r = 6$

$\therefore S'' > 0$, concave up

$\therefore$ minimum surface area occurs when $r = 6$

when $r = 6$, $h = \frac{360}{6^2} - \frac{2}{3} \times 6$

$h = 10 - 4$

$\therefore h = 6$.

$\therefore r = 6$ & $h = 6$.

Most of the student could differentiate S correctly, and find r.

Some students didn't show the test for nature of the stationary point

End of Paper